

Principles of

CLASSICAL.ELECTRODYNAMICS.pdf

A “laptop text”

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PREFACE

Preface to the handwritten edition of 1980/81. My *Mathematical Introduction to Electrodynamics* (1972/73) can be fairly described as the work of a “young formalist who should have known better;” it begins with a detailed account of special relativity, continues with detailed surveys of the elements of tensor analysis and the exterior calculus, and treats also the relativistic mechanics of particles before it arrives—after nearly 300 pages of preparation—at the dynamics of the electromagnetic field. Discussion even of the latter topic is marked by frequent “research digressions” of an invariably formalistic nature, digressions which I now consider to be (as even then I did) technically interesting but pedagogically extravagant. The frankly eccentric design of that earlier course can be partially understood if one takes into account the following circumstances:

- I was—by staff decision—under a formal obligation to teach both electrodynamics *and* the “methods of mathematical physics;”
- it was my explicit expectation that my students would be reading one or several of the standard texts collaterally;
- during the decade prior to 1972 Reed College physics students had shown a marked interest in formal/structural matters; the abrupt shift to a more “practical” set of interests and values first became conspicuous about 1973, and caught me by surprise (it anticipated a parallel shift in my own interest and values);
- I had never previously taught electrodynamics, and had “many rocks to turn over” in the service of my own technical education.

When repeated (1973/74 and 1974/75) the course was in fact less eccentric than those old notes suggest, for I omitted much of the formal material, and in its place treated radiation theory, as developed in my *Quantum Perturbations & Radiative Processes* (1973/74). But the formal emphasis and relative absence of phenomenological detail were still (in my present view) excessive.

The course design here recorded arose in part by reaction to that former teaching experience. My goal—as formerly—was not to “rewrite Jackson” but to expose as clearly as I could the structural heart of electrodynamics (and thus to *prepare my students to read* Jackson and similar texts). I was determined “to get Maxwell’s equations on the board” as soon as possible (I had recently reviewed a manuscript by Julian Schwinger which indicated how that goal might be accomplished), to treat “formal” problems only when the further elaboration of mainline electrodynamics made such activity unavoidable, and to abandon any explicit attempt to treat comprehensively the miscellaneous “methods of mathematical physics.” I wanted also to give relatively more attention to phenomenological matters, and to construct a written record of some of the things I had learned since 1975.

Here (as historically) relativity emerges *in natural consequence of a study of the transformation properties of Maxwell’s equations*, and attention is given to the little-known fact that a slight modification of that analytical program leads not to the Lorentz group but to the conformal group, the main properties of which are described in detail (details omitted when the course was repeated). Tensors are treated only in the detail specifically required, and the exterior calculus is (as is all reference to its electrodynamical applications) omitted altogether. The patient reader will still find too-frequent evidence of Wheeler’s compulsion to “turn over rocks,” and most readers will share my own judgment that the formalism is still too dense, and the reference to phenomenology still too slight. The latter defect was in practice somewhat blunted by the fact that students were encouraged to make heavy collateral use of David Griffiths’ *Introduction to Electrodynamics* (1981).

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Preface to the present electronic edition. This material came into being primarily because I had grown tired of late-night trips to Kinko's to print copies of my old hand-written notes for distribution in class, and of the attendant financial complications. I had become increasingly sensitive also to the circumstance that the material was in fact growing "old" (was already older than my students), and increasingly alert to the advantages of electronic publication, which had been impressed upon me by good experiences in several other courses. So in August 2001 I decided to produce a "revised electronic edition" of my 1980/81 class notes. I imagined the job would keep me busy until about November. In fact it absorbed my almost total attention over an eight-month period.

I found that I was, by and large, still fairly pleased with the basic design and execution of original text, but as I progressed the revisions became progressively more frequent, progressively more radical. Some of the original material has been boiled down or omitted altogether, analytical arguments have often been replaced with *Mathematica*-assisted "mathematical experiments," whether undertaken and reported by me or—at my request—by the students themselves. A fair amount of material (for example: everything having to do with conformal transformations and the covariance of Maxwell's equations), though retained, was omitted from the lectures.

On the other hand, some new material has been introduced. Most conspicuously (and eccentrically), I have allowed myself to draw upon elements of Proca's "theory of massive photons" in order to underscore certain critical respects in which classical electrodynamics is "atypical—poised on the razor's edge." And I have incorporated a theory of "optical beams as electromagnetic objects" that happened to occur to me as I wrote. During the interval 1981–2001 I had fairly frequent occasion to take up electromagnetic topics. *None* of that material was has been folded into these revised notes, though the substantial portion of it that existed already in electronic form was made available to students who cared to do some collateral reading.¹

It has been my lifelong experience that I learn most effectively not by close reading of what A has to say about the subject, or what B has to say, but by *comparing* A's and B's (and also C's) approaches to the same subject. It has been therefore not willful self-indulgence but something approaching a sense of duty that has led me to organize and approach the subject matter of electrodynamics in ways that many colleagues would consider eccentric. My presumption has been that my students will be comparing what I have to say with what Griffiths, Marion, Jackson, . . . have to say—this in their efforts to arrive at their own individual understandings of a complicated subject matter. My intent has been not to sing Griffiths' tune, but—because we are so fortunate as to have David Griffiths among us—to sing in a kind of *obbligato* harmony.

¹ I allude to "Electrodynamical application of the exterior calculus," (1996); "Algebraic theory of spherical harmonics," (1996); "'Electrodynamics' in 2-dimensional spacetime," (1997); "Simplified production of Dirac δ -function identities," (1997); "Theories of Maxwellian design," (1998).

This project began as an effort to solve a distribution problem, and to facilitate future editorial revision. But electronic publishing provides options not available in hard copy, so I soon confronted the question: “Am I generating material intended to be printed (in black and white) or to be read on-screen?” So great did I consider the advantages of using color to eliminate the distracting clutter of primes, superscripts and subscripts that—somewhat tentatively—I selected the latter option. Some information will therefore be lost when the text is laser-printed, but are led to believe that the cost/speed of ink-jet color printing will soon decrease/increase to realistic levels. Some students came to class with black & white hardcopy versions of the text, fewer with colored copy . . . and only one or two with their laptops. It is my hope and expectation that the latter practice will soon become the norm, for it belatedly occurred to me that what I have unwittingly produced is a “laptop text.” Once the general run of students become properly equipped (I yesterday made arrangements for the design of the prospective new physics lecture hall to be modified in anticipation of such a development) it will become possible to build animations, links to other documents—in short: the full range of electronic resources—into the design of a future edition of this and other texts.

The text was created with Textures[®] running T_EX on a PowerMac G-3 platform. The T_EX code was translated into PostScript by “printing to file,” and the final PDF (Portable Document Format) file was created by using Acrobat Distiller[®] to open the PostScript file (which was then discarded). Some of the figures were drawn by *Mathematica* and exported (to the Textures folder containing the T_EX code) as EPS files, others were drawn by hand using FreeHand8,[®] and some were created by using FreeHand8 to add details to *Mathematica* figures.

The PDF files, as distributed on the Courses Server, are all smaller—often *much* smaller—than the files from which they were created. They are intended to be opened and read with Acrobat Reader,[®] which is freeware distributed by Adobe. The Acrobat Reader is a powerful tool—capable of much more than simply opening PDF files—and readers are encouraged to familiarize themselves with its search, mark-up and other resources: the Visual QuickStart Guide *PDF with Acrobat* by Ted Alspach (1999) is very useful in this connection.

I am indebted to my students for their patience with a project which for the most part they seem to have taken entirely for granted (one suggested on a class evaluation form that the course might work much better if I adopted a better text), and especially to Eric Lawrence, who brought many typos and misspellings to my attention.

Nicholas Wheeler
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